Finite source single server retrial queueing model with control policy, balking and spares

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***Abstract*—In this paper, finite source single server retrial queueing model with control policy, balking and spares is considered. The steady state solutions and the system characteristics are derived and analyzed for this model. Numerical results are given for better understanding and relevant conclusion is presented**

***Keywords*— retrial queue; finite source; impatient customers; control policy; infinite capacity.**

INTRODUCTION

Retrial queueing systems differ from conventional queueing systems in that customers arriving to a server station and finding all servers unavailable enter a retrial orbit (or source of repeated calls) instead of a normal queue. Queueing systems in which primary customers who find all the servers and waiting positions (if any) occupied may retry for service after a long period of time are called retrial queues. Between retrials a customer is said to be in orbit. In general it is assumed that the arrival stream of primary calls, the service times and retrial times are mutually independent. Retrial queues have been widely used to model many problems in telephone switching systems, telecommunication networks and computer systems for competing to gain service from a central processing unit and so on. Moreover, retrial queues are also used as mathematical models for several computer systems: packet switching networks, shared bus local area networks operating under the carrier-sense multiple access protocol and collision avoidance star local area networks, etc. But the primary arrival and service processes are interdependent in practical situations. Balking occurs when a subscriber's call becomes rejected and the subscriber gets impatient and gives up after a certain time without getting service. The model studied in this paper not only takes into account retrials due to congestion but also considers the effects of balking and spares.

MODEL DESCRIPTION

Consider a single server infinite capacity finite source retrial queueing system in which primary customers arrive according to the Poisson flow of rate λ_1 and λ_2, service times are exponentially distributed with rate μ. If a primary customer finds some server free, he instantly occupies it and leaves the system after service. Otherwise, if the server is busy, at the time of arrival of a primary call then with probability $U \geq 0$ the arriving customer enters an orbit and repeats his demand after an exponential time with rate θ. Thus the Poisson flow of repeated call follow the retrial policy where the repetition times of each customer is assumed to be independent and exponentially distributed. If an incoming repeated call finds the line free, it is served and leaves the system after service, while the source which produced this repeated call disappears. Otherwise, if the server is occupied at the time of a repeated call arrival with probability (1-V) the source leaves the system without service.

It is assumed that the primary arrival process $[X_1(t)]$ and the service process $[X_2(t)]$ of the systems are correlated and follow a bivariate Poisson process given by

$$P(X_1 = x_1, X_2 = x_2; t) = e^{-(\lambda_i + \mu - \varepsilon)t} \frac{\sum_{j=0}^{\min(x_1 x_2)} \varepsilon(t)^j \; (\lambda_i - \varepsilon)t^{x_1 - j} \, (\mu - \varepsilon)t^{x_2 - j}}{j!\,(x_1 - j)!\,(x_2 - j)!}$$

$x_1, x_2 = 0,1,2,\ldots\ldots, \lambda_i , \mu < 0, \; i = 0,1;$

with parameters λ_1, λ_2, μ_n and ε as mean faster rate of primary arrivals, mean slower rate of primary arrivals, mean service rate and mean dependence rate (covariance between the primary arrival and service processes) respectively.

STEADY STATE EQUATION

Let $P_{0,n,0}$, $P_{0,n,1}$, $P_{1,n,0}$, $P_{1,n,1}$ denote the steady state probability that there are n customers in the queue when the system is in the faster and slower rate of primary arrivals and the server is idle and busy

The steady state equations are

$$-K(\lambda_0-\varepsilon)P_{0,0,0}+(\mu-\varepsilon)P_{1,0,0}=0 \qquad (1)$$

$$-[(K-1)U(\lambda_1-\varepsilon)+(\mu-\varepsilon)]P_{1,0,0}+K(\lambda_1-\varepsilon)P_{0,0,0}+\theta P_{0,1,0}+\theta V P_{1,1,0}=0 \qquad (2)$$

$$-[(K-n)(\lambda_1-\varepsilon)+n\theta]P_{0,n,0}+(\mu-\varepsilon)P_{1,n,0}=0 \qquad n=1,2,3,\ldots,R+1,R+2 \qquad (3)$$

$$-[(K-n-1)V(\lambda_1-\varepsilon)+(\mu-\varepsilon)+n\theta(1-V]P_{1,n,0}+[(K-n)(\lambda_1-\varepsilon)]P_{0,n,0}$$
$$+[(K-n)U(\lambda_1-\varepsilon)]P_{1,n-1,0}+(n+1)\theta P_{0,n+1,0}+[(n+1)\theta(1-V)]P_{1,n+1,0}=0,$$
$$n=1,2,3,\ldots,r-1 \qquad (4)$$

$$-[(K-r-1)U(\lambda_1-\varepsilon)+(\mu-\varepsilon)+r\theta(1-V)]P_{1,r,0}+[(K-r)(\lambda_1-\varepsilon)]P_{0,r,0}+[(N-r)U(\lambda_1-\varepsilon)]P_{1,r-1,0}$$
$$+(r+1)\theta P_{0,r+1,0}+[(r+1)\theta(1-V)]P_{1,r+1,0}+(r+1)\theta P_{0,r+1,1}+[(r+1)\theta(1-V)]P_{1,r+1,1}=0 \qquad (5)$$

$$-[(-[(K-n-1)U(\lambda_1-\varepsilon)+(\mu-\varepsilon)+n\theta(1-V)]P_{1,n,0}+[(K-n)(\lambda_1-\varepsilon)]P_{0,n,0}+[(K-n)U(\lambda_1-\varepsilon)]P_{1,n-1,0}$$
$$+(n+1)\theta P_{0,n+1,0}+[(n+1)\theta(1-V)]P_{1,n+1,0}=0,$$
$$n=r+1,r+2,\ldots\ldots,R-2 \qquad (6)$$

$$-[(K-R)U(\lambda_1-\varepsilon)+(\mu-\varepsilon)+(R-1)\theta(1-V)]P_{1,R-1,0}+[(K-R+1)(\lambda_1-\varepsilon)]P_{0,R-1,0}$$
$$+[(K-R+1)U(\lambda_1-\varepsilon)]P_{1,R-2,0}=0 \qquad (7)$$

$$-[(K-r-2)U(\lambda_2-\varepsilon)+(\mu-\varepsilon)+(r+1)\theta(1-V)]P_{1,r+1,1}+[(K-r-1)(\lambda_2-\varepsilon)]P_{0,r+1,1}+$$
$$[(K-n)U(\lambda_2-\varepsilon)]P_{0,r+1,1}+(r+2)\theta P_{0,r+2,1}+[(r+2)\theta(1-V)]P_{1,r+2,1}=0 \qquad (8)$$

$$-[(K-n-1)U(\lambda_2-\varepsilon)+(\mu-\varepsilon)+n\theta(1-V)]P_{1,n,1}+[(K-n)(\lambda_2-\varepsilon)]P_{0,n,1}+$$
$$[(K-n)U(\lambda_2-\varepsilon)]P_{1,n-1,1}+(n+1)\theta P_{0,n+1,1}+[(n+1)\theta(1-V)]P_{1,n+1,1}=0,$$
$$n=r+2,r+3,\ldots\ldots.R-1 \qquad (9)$$

$$-[(K-R-1)U(\lambda_2-\varepsilon)+(\mu-\varepsilon)+R\theta(1-V)]P_{1,R,1}+[(K-R)(\lambda_2-\varepsilon)]P_{0,R,1}$$
$$+[(K-R)U(\lambda_2-\varepsilon)]P_{1,R-1,1}+[(K-R)U(\lambda_2-\varepsilon)]P_{1,R-1,0}+(R+1)\theta P_{0,R+1,1}$$
$$+[(R+1)\theta(1-V)]P_{1,R+1,1}=0 \qquad (10)$$

$$-[(K-n-1)U(\lambda_2-\varepsilon)+(\mu-\varepsilon)+n\theta(1-V)+(n-1)]P_{1,n,1}+[(K-n)(\lambda_2-\varepsilon)]P_{0,n,1}+$$
$$[(K-n)U(\lambda_2-\varepsilon)]P_{1,n-1,1}+(n+1)\theta P_{0,n+1,1}+[(n+1)\theta(1-V)]P_{1,n+1,1}=0,$$
$$n=R+1,R+2,\ldots\ldots\ldots \qquad (11)$$

Write $S_1=[U(\lambda_1-\varepsilon)]$ and $S_2=[U(\lambda_2-\varepsilon)]$

From (1) to (4) we get,

$$P_{0,n,0}=\frac{(K-1)_n {S_1}^n\prod_{i=0}^{n-1}[(K-i)(\lambda_1-\varepsilon)+i\theta]}{\prod_{i=1}^{n} i\theta(\mu-\varepsilon)+[i\theta(1-V)][(K-i)(\lambda_1-\varepsilon)+i\theta]}P_{0,0,0} \qquad 1\le n\le r \qquad (12)$$

$$P_{1,n,0}=\frac{(K-1)_n {S_1}^n}{\mu-\varepsilon}\prod_{i=1}^{n}\frac{[(K-i)(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-V)][(K-i)(\lambda_0-\varepsilon)+i\theta]}\quad P_{0,0,0} \qquad (13)$$

Using (3),(5) and (6) we get,

$$P_{0,n,0}=\frac{(K-1)_n {S_1}^n \prod_{i=0}^{n-1}[(K-i)(\lambda_1-\varepsilon)+i\theta]}{\prod_{i=1}^{n} i\theta(\mu-\varepsilon)+[i\theta(1-V)][(K-i)(\lambda_1-\varepsilon)+i\theta]}P_{0,0,0}$$

$$-\left\{\frac{A_1}{A_2}\left(\left[\sum_{m-r}^{n-2}(K-m-)S_1^{\,n-1-m}\quad\prod_{i=m+1}^{n}\frac{[(K-i)(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-V)][(K-i)(\lambda_1-\varepsilon)+i\theta]}\right]+\right)\right\}\quad P_{0,r+1,1} \qquad (14)$$

$$P_{1,n,0}=\frac{(K-1)_n {S_1}^n}{\mu-\varepsilon}\prod_{i=1}^{n}\frac{[(K-i)(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-V)][(K-i)(\lambda_1-\varepsilon)+i\theta]}\qquad P_{0,0,0}$$

$$-\left\{\frac{A_1}{A_2(\mu-\varepsilon)}\left(\left[\sum_{m-r}^{n-1}(K-m-2)S_1^{\,n-1-m}\quad\prod_{i=m+1}^{n}\frac{[(K-i)(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-V)][(K-i)(\lambda_1-\varepsilon)+i\theta]}\right]\right)\right\}P_{0,r+1,1}$$

n = r+1,r+2,……,R-1 (15)

where

$A_1 = (r+1)\theta(\mu-\varepsilon)+[(r+1)\theta(1-V)][(K-r-1)(\lambda_1-\varepsilon)+(r+1)\theta]$

$A_2 = n\theta(\mu-\varepsilon)+[n\theta(1-V)][(K-n)(\lambda_1-\varepsilon)+n\theta]$

Using (3) & (7) we get, $P_{0,r+1,1}=\frac{A_3}{A_4}p_{0,0,0}$

Where $A_3=\frac{(K-1)_R {T_1}^R \prod_{i=0}^{R-1}[(K-i)(\lambda_1-\varepsilon)+i\theta]}{\prod_{i=1}^{R-1} i\theta(\mu-\varepsilon)+[i\theta(1-V)][(K-i)(\lambda_1-\varepsilon)+i\theta]}P_{0,0,0}$

$$A_4=-\left\{A_1\left(\left[\sum_{m-r}^{R-2}(K-m-2)S_1^{\,n-1-m}\prod_{i=m+1}^{R-1}\frac{[(K-i)(\lambda_1-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-V)][(K-i)(\lambda_1-\varepsilon)+i\theta]}\right]\right)\right\}P_{0,r+1,1} \qquad (16)$$

From (3),(8) & (9), we recursively derive,

$$P_{0,n,1}=\left\{\frac{A_1}{A_5}\left(\left[\sum_{m-r}^{n-2}(K-m-2)S_2^{\,n-1-m}\prod_{i=m+1}^{n-1}\frac{[(K-i)(\lambda_2-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-V)][(K-i)(\lambda_1-\varepsilon)+i\theta]}\right]+1\right)\right\}\quad P_{0,r+1,1} \qquad (17)$$

$$P_{1,n,1}=\left\{\frac{A_1}{\mu-\varepsilon}\left(\left[\sum_{m-r}^{n-1}(K-m-2)S_2^{\,n-1-m}\prod_{i=m+1}^{n}\frac{[(K-i)(\lambda_2-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-V)][(K-i)(\lambda_1-\varepsilon)+i\theta]}\right]\right)\right\}P_{0,r+1,1}$$

n= r+1,r+2,……R-1,R (18)

where

$A_5 = n\theta(\mu-\varepsilon) + [n\theta(1-V)][(K-n)(\lambda_2-\varepsilon)+n\theta]$,A_1 is given by (15) and $P_{0,r+1,1}$ is given by (16)

From (3),(10) &(11) we recursively derive,

$$P_{0,n,1} = \left\{\frac{A_1}{A_5}\left(\left[\sum_{m-r}^{R-2}(K-m-2){S_2}^{n-1-m}\prod_{i=m+1}^{n-1}\frac{[(K-i)(\lambda_2-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-V)][(K-i)(\lambda_1-\varepsilon)+i\theta]}\right]\right)\right\}P_{0,r+1,1} \tag{19}$$

$$P_{1,n,1} = \left\{\frac{A_1}{\mu-\varepsilon}\left(\left[\sum_{m-r}^{R-1}(K-m-2){S_2}^{n-1-m}\prod_{i=m+1}^{n}\frac{[(K-i)(\lambda_2-\varepsilon)+i\theta]}{i\theta(\mu-\varepsilon)+[i\theta(1-V)][(K-i)(\lambda_1-\varepsilon)+i\theta]}\right]\right)\right\}P_{0,r+1,1}$$

n=R+1, R+2,……… (20)

where A_1, A_5 and $P_{0,r+1,1}$ are given by (14), (16), (17).

Thus from (12) to (20), we find that all the steady state probabilities are expressed in terms of $P_{0,0,0}$.

CHARACTERISTICS OF THE MODEL

The following system characteristics are considered and their analytical results are derived in this system.

- The probability P(0), P(1) that the system is in faster and slower rate of primary arrivals with the server idle and busy.The probability $P_{0,0,0}$ that the system is empty.The expected number of customers in the system Ls_0, Ls_1, when the system is in faster and slower rate of primary arrivals with the server idle and busy.
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The probability that the system is in faster rate of primary arrivals is

$$P(0) = \left[\sum_{n=0}^{r} P_{0,n,0} + \sum_{n=r+1}^{R-1} P_{0,n,0}\right] + \left[\sum_{n=0}^{r} P_{1,n,0} + \sum_{n=r+1}^{R-1} P_{1,n,0}\right] \tag{21}$$

The probability that the system is in slower rate of primary arrivals is,

$$P(1) = \left[\sum_{n=r+1}^{R} P_{0,n,1} + \sum_{n=R+1}^{K} P_{0,n,1}\right] + \left[\sum_{n=r+1}^{R} P_{1,n,1} + \sum_{n=R+1}^{K} P_{1,n,1}\right] \tag{22}$$

The probability $P_{0,0,0}$ that the system is empty can be calculated from the normalizing condition
P(0) + P(1) =1. $P_{0,0,0}$ is calculated from (21) and (22) .

Let L_s denote the average number of customers in the system, then we have

$$L_s = \left[\sum_{n=0}^{r} nP_{0,n,0} + \sum_{n=r+1}^{R-1} nP_{0,n,0}\right] + \left[\sum_{n=0}^{r}(n+1)P_{1,n,0} + \sum_{n=r+1}^{R-1}(n+1)P_{1,n,0}\right] + \left[\sum_{n=r+1}^{R} nP_{0,n,1} + \sum_{n=R+1}^{K} nP_{0,n,1}\right] + \left[\sum_{n=r+1}^{R}(n+1)P_{1,n,1} + \sum_{n=R+1}^{K}(n+1)P_{1,n,1}\right] \tag{23}$$

The expected waiting time of the customers in the orbit is calculated as $W_s = \frac{L_s}{\bar{\lambda}}$, Where $\bar{\lambda} = \lambda_0$ P(0) + λ_1P(1). W_s is calculated from (21) to (23).

NUMERICAL ILLUSTRATIONS

For various values λ_1, λ_2, μ, ϵ, θ, K while r, R, K, U, V are fixed values, computed and tabulated the values of $P_{0,0,0}$,P(0), P(1), L_s and W_s.

TABLE

r	R	λ_1	λ_2	μ	θ	U	V	ϵ	K	$P_{0,0,0}$	P(0)	P(1)	L_s	W_s
3	6	2	1	3	1	0.8	0.2	0.5	11	3.688×10^{-3}	0.3890	0.8239	6.1975	3.4191
3	6	3	1	3	1	0.8	0.2	0.5	11	7.235×10^{-4}	0.1592	0.9811	5.6786	2.8579
3	6	4	2	3	1	0.8	0.2	0.5	11	6.167×10^{-4}	0.1588	0.9823	6.2397	2.2181
3	6	3	2	3	2	0.8	0.2	0.5	11	3.713×10^{-3}	0.2439	0.9872	3.3094	0.9980
3	6	3	2	3	2	0.8	0.2	0.5	12	8.527×10^{-4}	0.3285	0.7946	5.3628	1.7558
3	6	3	2	4	2	0.8	0.2	0.5	11	3.328×10^{-3}	0.5513	0.5798	2.8897	0.8751
3	6	3	2	3	1	0.8	0.2	1	11	0.565×10^{-3}	0.2699	0.9806	6.1478	2.9754
3	6	3	2	3	2	0.8	0.2	0.5	11	1.756×10^{-3}	0.4360	0.7948	3.1965	0.9781
3	6	2	2	3	1	0.8	0.2	0.5	11	1.7735×10^{-3}	0.3847	0.8452	6.2141	2.8397

CONCLUSION

From the table it is observed that when λ_1,λ_2 , K increases keeping the other parameters fixed, $P_{0,0,0}$ and P(0) decrease but P(1), Ls and Ws increase. When θ , μ decrease keeping the other parameters fixed, $P_{0,0,0}$ and P(0) decrease but P(1), Ls and Ws increase

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