Limited failure censored life test sampling plan in Log-Logistic Distribution

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ABSTRACT

This paper deals with a log-logistic distribution as a life time model that a product in a submitted lot for acceptance or otherwise is assumed to follow log-logistic distribution with β =3. The decision making process is based on the inspection of a sample of items for this life times taken from the lot such a sample is divided into groups to develop a group sampling plan with a mechanism of terminating life testing process as soon as the first failure in each group is noticed. The criterion for accepting submitted lot is proposed.

Keywords: single sampling, lot acceptance, group sampling plan, truncated life test, reliability test plans, order statistics.

Introduction:

In classical statistics the acceptance sampling plans play an important role. Here acceptance sampling is concerned with inspection and decision making regarding products. If the quality of a product is measured through the life time, the sampling plans to determine acceptability of a product with respect to life time are called Reliability sampling plans. When the life time random variable is assumed to follow a continuous probability distribution, sampling plans are developed by various researchers covered a wide spectrum of probability models. Most of these works are referred in Kantam & Ravi Kumar (2016). In all these words, given the termination time of a life test, the construction of a sampling plan consists of determining the minimum number of sample items that are to be life tested and the acceptance number beyond which the observed failures out of the life test items of the sample lead to rejection of the submitted lot, condition on pre specified producer's and consumer's risk. In this sequel Kantam & Ravi Kumar (2016) proposed a new sampling plan called Limited failure censored Life test sampling plan (LFCLTSP). For Dagum distribution similar work is done by Srinivasa Rao *et al.*(2018) and Ravi Kumar *et al.*(2019). Infact, this LFCLTSP is an alternative criterion proposed by Jun *et al.*(2006). Weibull probability model by Jun *et al.* (2006) and Kantam & Ravi Kumar (2016) for Burr type X distribution and both are popular life testing models. This scheme of life testing and termination process is named by some researchers as *Sudden death Testing* (for example Pascual & Meeker (1998); Jun *et al.*(2006)). The name 'Limited failure censored life tests is proposed by Wu *et al.* on the lines of Kantam & Ravi Kumar (2016) LFCLTSP is developed for the Log-Logistic Distribution for β = 3 in this paper. In Section 2 construction of LFCLTSP for Log-Logistic distribution β = 3 is presented and the results are explained in Section3.

2. Construction of LFCLTSP for Log-Logistic Distribution

Let $Y_1, Y_2, \ldots, Y_m$ are m first ordered statistics in the limited failures censored sample of m independent random samples of size n each. If Z denotes the maximum of $Y_1, Y_2, \ldots, Y_m$ which may also be viewed as the total test time/experimental time by Kantam and Srinivasa Rao (2004). If Z is larger realised value can be considered as indication that the products in the submitted lot have longer life prompting one to consider the lot as a good lot for acceptability which can be "Z > CL" can be taken as criterion for acceptance of the lot. Thus Kantam & Ravi Kumar (2016) proposed the following decision rule.

1. Draw a random sample of size N= mxn and allocate n items to each m groups.
2. Observe Y_i the time to first failure in the i^{th} group (i=1,2,…,m).
3. Identify the quantity $Z = Max(Y_1, Y_2, \ldots, Y_m)$.
4. Accept the lot of Z ≥ CL and reject the lot otherwise (c may be called the acceptability constant – a concept similar to the acceptance number in time truncated reliability test plans).

Using the theory of ordered statistics we can get the cumulative distribution function of Z in a closed form as long as the cumulative distribution function of the base line distribution in a closed form hence the percentiles of Z can be used to get the designed parameters m, c analytically. The following is the analytical procedure for calculating designed parameters of LFCLTSP for our log logistic distribution (β = 3). The probability density function (pdf) of Log logistic distribution (β = 3) is given by

$$f(z) = \frac{\beta z^{\beta-1}}{(1+z^{\beta})^2}, 0 \le z < \infty, \beta = 3 \quad (2.1)$$

Cumulative distribution function of Log logistic distribution is

$$F(z) = \frac{z^3}{(1+z^3)}, 0 \le z < \infty, \beta = 3 \quad (2.2)$$

The fraction non-conforming or unreliability is expressed by

$$p = Pr(X < L) = F(L) \quad (2.3)$$

If p is given by the corresponding L is obtained from

$$w = L = \sqrt[3]{p/(1-p)} \quad (2.4)$$

Let $X_1, X_2, \ldots, X_n$ be a random sample of size 'n' from (2.2), The Cumulative distribution function of $X_1, X_2, \ldots, X_n$ is given by

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$$F_{(1)(x))} = 1 - 1 - [1 - F(x)]^n \; = [1 - F(x)]^n \qquad (2.5)$$

$$F_{(1(x))} = 1 - \frac{1}{(1+x^3)^n} \qquad (2.6)$$

$\therefore$ $Y_{1\,,}Y_{2\,,}\ldots.Y_m$ of the limited failure censored test are now a random sample of size m from $F_{(1(x))}$ hence the cumulative distribution function of Z the largest of $Y_1, Y_2,\ldots,Y_m$ is given by

$$G_m(z) = \; F_{1(z)^m} \qquad (2.7) \quad \text{i.e.,}$$

$$G_m(z) = \left(1 - \frac{1}{(1+x^3)}\right)^m \qquad (2.8)$$

The designed parameter m and c of LFCLRSP are obtained with the help of percentiles of $G_m(z)$ is given in (2.8) of α and β are respectively, the producer's and consumer's risks for desirable/acceptance lot quantity level p. Undesirable/ lot tolerance quantity level p, then m and c are the solutions of the following two inequalities.

$$G_m(cw_0) \;\leq\; \alpha \qquad (2.9)$$

$$G_m(cw_1) \geq 1 - \beta \qquad (2.10)$$

where w_0 and w_1 are the solution of (2.4).

The inequalities (2.7), (2.8) respectively implies

$$cw_0 \leq G_m^{-1}\,(1 - \alpha) \qquad (2.11)$$

$$cw_1 \leq G_m^{-1}\,(\beta) \qquad (2.12)$$

which reduces to $^{w_0}/_{w_1} \;\leq\; \frac{G_m^{-1}\,(1-\alpha)}{G_m^{-1}\,(\beta)}$ (2.13)

$\therefore$ m can be obtained by the smallest integer satisfying (2.13). The acceptability constant c can be obtained from the equality case in either of the expression (2.11), (2.12). We have tabulated the values of m and c which are analytically determined for the selecting combinations of p_0 and p_1 and is presented in table (2.1). The values of m obtained by LFCLTSP can be seen to be consistently smaller, Thus this sampling plan indicating less number of items to be put to life test.

Table – 2.1: Design Parameters of LFCLTSP for LLD at α=0.05 and β=0.1

(Min-Max) Approach for **LLD** at α=0.05 and β=0.1					
p_0	p_1	$\boldsymbol{m}$		$\boldsymbol{c}$	
		$\boldsymbol{n=5}$	$\boldsymbol{n=10}$	$\boldsymbol{n=5}$	$\boldsymbol{n=10}$
0.005	0.02	16	9	4.39	2.99
	0.04	3	3	2.68	2.11

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	0.06	2	2	2.18	1.72
	0.08	2	2	2.18	1.72
	0.10	2	2	2.18	1.72
	0.14	2	2	2.18	1.72
	0.20	2	2	2.18	1.72
0.01	0.04	15	9	3.43	2.37
	0.06	5	4	2.58	1.87
	0.08	3	2	2.12	1.36
	0.10	2	2	1.73	1.36
	0.14	2	2	1.73	1.36
	0.20	2	2	1.73	1.36
0.02	0.08	13	8	2.63	1.82
	0.10	7	5	2.26	1.60
	0.14	4	3	1.88	1.32
	0.20	2	2	1.37	1.08
0.03	0.14	8	5	2.04	1.39
	0.20	3	3	1.46	1.15
0.04	0.20	6	4	1.70	1.17
0.05	0.20	10	6	1.81	1.23

3. EXAMPLE:

The quality assurance in a bearing manufacturing process states that p_0=0.03, p_1=0.20, α=0.05, β=0.1 the number of test positions (size of each group, *n*)=10. For this information Table – 2.1 of suggests *m*=3, *c*=1.15. Accordingly a random sample of size *N*=50 items are put to test in five groups with 10 items in each group. The observed first failure times in the five groups are Y_1=120, Y_2 =200, Y_3=185, Y_4=55, Y_5=265. Assuming that the life times follow Log Logistic distribution and a lower specification of L=100 they have at the above p_0, p_1, α, β, *n*=10, and acceptability constant *c*=1.15 then cL =115. *Z*= The minimum failure of maximum of five groups is 55. Since $Z < cL$. *i.e.*, 55 < 115, the lot is to be rejected.

From this example, we see that our approach reached the decision of rejecting the lot by conducting limited failure censored life test for only three groups of 10 items each, resulting in low cost of experimentation and lower number of destructions.

More over it may be recalled that Z are defined as $Z = Max(Y_1, Y_2, ..., Y_m)$. If *c* is the acceptability constant and *L* is the lower specification, $Z > cL$. That is acceptance decision of LFCLTSP is considered and gives a stronger conclusion with this illustration.

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