THE DUSTY FLUID'S BOUNDARY LAYER FLOW AND CONVECTIVE HEAT TRANSFER OVER A STRETCHING SHEET WITH THERMAL RADIATION ARE AFFECTED BY PARTICLES' ELECTRIFICATION

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Abstract

This study focuses on the continuous free convective flow of a dusty fluid across a vertical permeable stretching surface containing electrified particles. Using similarity transformations, all partial differential equations are transformed into first order ordinary differential equations. Runge-Kutta fourth order method and firing technique are used to numerically solve these ordinary differential equations. Physical parameters such as the fluid-particle interaction parameter, the local Grashof number, the suction parameter, the Prandtl number, the radiation parameter, and the Eckert number are computed, and the effects on the flow and heat transfer characteristics are presented graphically and in tabular form. As the value of the electrification parameter increases, the rate of heat transmission at the surface and skin friction both rise.

Key words; Electrification of particle , Thermal radiation, Volume fraction, Interaction parameter, Dusty fluid, , Suction parameter, steady flow and heat transfer, Boundary layer flow.

Nomenclature

E_c Eckert number

q_r radiation heat flux

q_{r_p} radiation heat flux of particle phase

F_r Froud number

G_r Grashof number

P_r Prandtl number

T_∞ temperature at large distance from the wall.

T_p temperature of particle phase.

T_w wall temperature

$U_w(x)$ stretching sheet velocity

c_p specific heat of fluid

c_s specific heat of particles

k_s thermal conductivity of particle

u_p , v_p velocity component of the particle along x-axis and y-axis

A constant

Ra Thermal radiation

c stretching rate

f_0 suction parameter

g acceleration due to gravity

k thermal conductivity of fluid

l characterstic length

T temperature of fluid phase.

u,v velocity component of fluid along x-axis and y-axis

x,y cartesian coordinate

K* Mean absorption co-efficient

Greek Symbols :

φ volume fraction

β fluid particle interaction parameter

β^* volumetric coefficient of thermal expansion

σ^* the Stefan Bolzman constant

ρ density of the fluid

ρ_p density of the particle phase

ρ_s material density

η similarity variable

θ fluid phase temperature

θ_p dust phase temperature

μ dynamic viscosity of fluid

ν kinematic viscosity of fluid

γ ratio of specific heat

τ relaxation time of particle phase

τ_T thermal relaxation time i.e. the time required by the dust particle to adjust its temperature relative to the fluid.

τ_p velocity relaxation time i.e. the time required by the dust particle to adjust its velocity relative to the fluid.

ε diffusion parameter

ω density ratio

Introduction

Sakiadis [] was the first to study the behaviour of boundary layer flow over a continuously moving flat surface submerged in an otherwise quiescent fluid theoretically using both precise and approximative approaches. Tsou et al. [] expanded the same study to the heat transport problem and experimentally validated the analysis of [1]..Crane [] extended the work of Sakiadis[] to a stretching sheet with linear surface velocity and obtain a similarity solution to the problem .Since then ,research area of stretching sheet has been flooded with many research articles with multiple dimensions enriched by the innovative researchers [ ].In summery the various concepts of the phenomenon are ,heat and mass transfer on horizontal/vertical plates , on inclined plates ,with or without suction or blowing ,steady flow, unsteady flow due to sudden stretching of sheet or by changing the temperature of the sheet, wall temperature, magnetic field, effect of diffusion –thermo and thermal diffusion of heat, (Soret and Dufour effect) ,uniform/non-uniform heat source/sink, thermal radiation, heat transfer over a porous stretching surface, flow through porous media. All the above investigations restricted their analysis to the flow induced by a linear/vertical stretching sheet under different physical situations and in the absence of fluid particle suspensions.

It is worth mentioning here that the two phase flows, in which solid spherical particles distributed in a fluid ,are of interest in a wide range of technical problems ,such as flow through packed beds, sedimentation, environmental pollution , nuclear reactor cooling, powder technology, rain erosion ,paint spraying, centrifugal separation, combustion and purification of crude oil , flowing rockets and blood rheology etc. The study of the boundary layer of fluid- particle suspension flow is important in determining the particle accumulation and impingement of particles on the surface [ ].To date an enormous amount of work has been done on the boundary layer flow and heat transfer with consideration of the stretching sheet problem [ ].The engineering applications of the stretching sheet problems includes polymer sheet extrusion from a dye, drawing ,tinning and annealing of copper wires ,glass fiber and paper production ,the cooling of metallic plate in a cooling bath and so on. A quick review of the above mentioned literature shows that the investigation is based upon the various physical concepts already told above. No consulted effort or hardly any attempt has been made to show the effect of electrification of particles and/or in contribution of various physical aspects .Since tribo electrification occurs due to collinear of particles with each other or impingement of particles with walls and since the electrification of particles have a pronounced effect on boundary layer characteristics like such friction, heat transfer etc, it is essential to include this phenomena in the modeling of flow over a stretching sheet .Even though the study relating to flow and heat transfer in MHD dusty boundary layer flow over stretching sheet [ ]are available, hardly any study is taken up by considering the base fluid as non –conducting and the particles are electrified. The forces and moments acting on a solid particle consist of those due to the net charge in the electric field due to the charged particles. As a general statement, any volume element of charge species, with charge "e" experiences an instantaneous force given by the Lorentz force law given by $\overrightarrow{f} = e\,\vec{E} + \overrightarrow{J} \times \vec{B}$ where $\vec{B}$ is the magnetic flux density. The current densities in corona discharge are so low that the magnetic force term $\overrightarrow{J} \times \vec{B}$ can be omitted, as this term is many orders of magnitude smaller than the Coulomb term$e\vec{E}$.The ion drift motion arises from the interaction of ions, constantly subject to the Lorentz force with the dense neutral fluid medium. This interaction produces an effective drag force on the ions. The drag force is in equilibrium with the Lorentz force so that the ion velocity in a field $\vec{E}$ is limited to $k_m\vec{E}$, where k_m is the mobility of the ion species. The drag force on the ions has an equal and opposite reaction force acting on the neutral fluid molecules via this ion-neutral molecules interaction, the force on the ions is transmitted directly to the fluid medium, so the force on the fluid particles is also given by $\overrightarrow{f} = e\,\vec{E}$. Soo [19]

The above analyses motivated to present study of the present paper. Here the particles will be allowed to diffuse through the carrier fluid i.e. the random motion of the particles shall be taken into account because of the small size of the particles. This can be done by applying the kinetic theory of gases and hence the motion of the particles across the streamline due to the concentration and pressure diffusion. We have considered the terms related to the heat added to the system to slip-energy flux in the energy equation of particle phase. The momentum equation for particulate phase in normal direction, heat due to conduction and viscous dissipation in the energy equation of the particle phase have been considered for better understanding of the boundary layer characteristics. The effects of electrification, radiation effect, and volume fraction of particles on skin friction, heat transfer and other boundary layer characteristics also have been studied.

In the present paper, the behavior of incompressible, laminar boundary- layer flows of a dusty fluid over a permeable vertical stretching sheet in presence of electrification of particles. To the author’s knowledge no consulted effort has been made to show the effect of electrification of particles along with particle and particle interaction as well as radiation effect. Electrification of solid particles occurs because of impact with the wall at low temperature. The governing equation are reduced into system of ODE and solved them using well known Runge Kutta Fourth order method and shooting technique. We have considered the terms related to the heat added to the system to slip-energy flux in the energy equation of particle phase. The effects of volume fraction on skin friction, heat transfer and other boundary layer characteristics also have been studied.

Flow Analysis of the Problem and solution:-

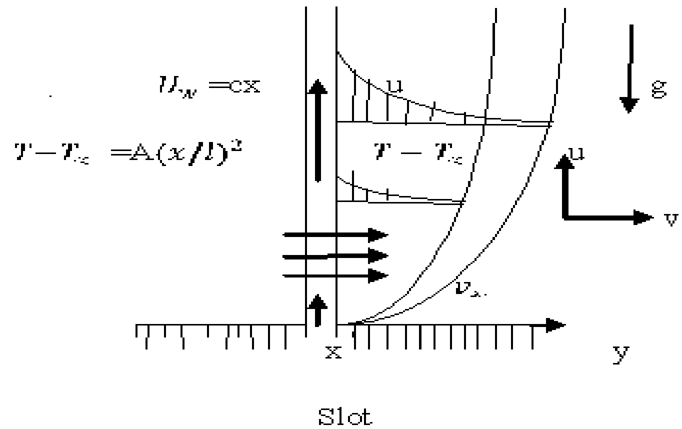


Figure-1 Schematic diagram of the flow

Consider a steady two dimensional laminar boundary layer of an incompressible viscous dusty fluid over a vertical stretching sheet .The flow is generated by the action of two equal and opposite forces along the x-axis and y-axis being normal to the flow .The sheet being stretched with the velocity $U_w(x)$ along the x-axis, keeping the origin fixed in the fluid of ambient temperatureT_∞. Both the fluid and the dust particle clouds are suppose to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size and number density of the dust particle is taken as a constant throughout the flow.

The governing equations of steady two dimensional boundary layer incompressible flows of dusty fluids are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial}{\partial x}(\rho_p u_p)+\frac{\partial}{\partial y}(\rho_p v_p)=0 \qquad (2)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho}\,\mu\frac{\partial^2 u}{\partial y^2}-\frac{1}{(1-\varphi)\rho}\frac{1}{\tau_p}\varphi\rho_s(u-u_p)+g\beta^*(T-T_\infty)+\frac{1}{1-\varphi}\frac{\rho_p}{\rho}\left(\frac{e}{m}\right)E \qquad (3)$$

$$\varphi\rho_s\left(u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial u_p}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s(u-u_p)+\varphi(\rho_s-\rho)g+\rho_p\left(\frac{e}{m}\right)E \qquad (4)$$

$$\varphi\rho_s\left(u_p\frac{\partial v_p}{\partial x}+v_p\frac{\partial v_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial v_p}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s(v-v_p) \qquad (5)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\varphi\rho_s c_s}{(1-\varphi)c_p}\frac{1}{\rho\tau_T}(T_p-T)+\frac{\varphi\rho_s}{(1-\varphi)}\frac{1}{\rho c_p}\frac{1}{\tau_p}(u_p-u)^2+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+$$
$$\frac{1}{1-\varphi}\frac{1}{\rho c_p}\rho_p\left(\frac{e}{m}\right)EU_p \qquad (6)$$

$$u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}=-\frac{1}{\tau_p}(T_p-T)+\frac{1}{\varphi\rho_s c_s}\frac{\partial}{\partial y}\left(\varphi k_s\frac{\partial T_p}{\partial y}\right)-\frac{1}{\tau_p}\frac{1}{c_s}(u-u_p)^2+\frac{\mu_s}{\rho_s c_s}\left[u_p\frac{\partial^2 u_p}{\partial y^2}+\left(\frac{\partial u_p}{\partial y}\right)^2\right]-$$
$$\varphi\frac{\partial q_{rp}}{\partial y}+\rho_p\left(\frac{e}{m}\right)EU_p \qquad (7)$$

Where (u,v) *and* (u_p,v_p) are the velocity components of the fluid and dust particle phases along x and y directions respectfully. μ , ρ and ρ_p, N are the co-efficient of viscosity of the fluid, density of the fluid and particle phase, number density of the particle phase respectfully.

With boundary conditions

$$u = U_\omega(x) = cx\,,v = -v_\omega(x)\,T = T_w = T_\infty + A\left(\frac{x}{l}\right)^2 \;\; at\; y = 0$$
$$\rho_p = \omega\rho, u = 0\,, u_p = 0\,, v_p \to v\,, T \to T_\infty\;\;, T_p \to T_\infty\;\; as\; y \to \infty \tag{8}$$

Where ω is the density ratio in the main stream and A is a positive constant,

$l = \sqrt{\frac{\nu}{c}}$ is a characteristic length.

Using the Rosseland approximation for radiation heat flux is simplified as

$$q_r = \frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial y} \tag{9}$$

Where σ^* and K^* are the Stefan Bolzman constant and the mean absorption co-efficient respectfully. Assuming that the temperature differences within the flow such that term T^4 may be expressed as a linear function of the temperature. We expand T^4 in a Taylor series about T_∞ and neglecting the higher order terms beyond the first degree in $(T - T_\infty)$we get

$$T^4 \cong 4T_\infty^4 \text{T-} 3T_\infty^4 \tag{10}$$

For most of the gases $\tau_p \approx \tau_T$, $k_s = k\frac{c_s}{c_p}\frac{\mu_s}{\mu}$ if $\frac{c_s}{c_p} = \frac{2}{3P_r}$, $\varphi\rho_s = \rho_p$ Introducing the following non dimensional variables in equation (1) to (7)

$$u = cxf'(\eta)\,, v = -\sqrt{c\nu}f(\eta)\,, \eta = \sqrt{\frac{c}{\nu}}\,y\,, u_p = cxF'(\eta)\,, v_p = \sqrt{c\nu}\,G(\eta)\,, \varphi\rho_r = H(\eta)$$
$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}\;,\; \theta_p(\eta) = \frac{T_p-T_\infty}{T_w-T_\infty}\,, \beta = \frac{1}{c\tau_p}\,, \epsilon = \frac{v_s}{\nu}\,, P_r = \frac{\mu c_p}{k}\;, E_c = \frac{c^2l^2}{Ac_p}, R_a \;\; = \frac{16T_\infty^3\sigma^*}{3K^*k},$$
$$\frac{\partial q_{r_p}}{\partial y} = -\frac{16T_\infty^3\sigma^*}{3K^*}\frac{\partial^2 T_p}{\partial y^2}, \text{M} = \frac{E}{c^2x}\left(\frac{e}{m}\right) \text{ Where } T - T_\infty = A\left(\frac{x}{l}\right)^2\theta\,, T_p - T_\infty = A\left(\frac{x}{l}\right)^2\theta_p \tag{11}$$

C is the stretching rate and being a positive constant, c_p is the specific heat of fluid phase.
K is the thermal conductivity, β is the fluid particle interaction parameter.
β^* is the volumetric coefficient of thermal expansion. We get the following non dimensional form.

$$HF + HG' + GH' = 0 \tag{12}$$
$$f'''(\eta) + f(\eta)f''(\eta) - [f'(\eta)]^2 + \frac{1}{(1-\varphi)}\beta\, H(\eta)[F(\eta) - f'(\eta)] + Gr\theta + \frac{H(\eta)}{1-\varphi}M = 0 \tag{13}$$
$$G(\eta)F'(\eta) + [F(\eta)]^2 = \epsilon\, F''(\eta) + \beta[f'(\eta) - F(\eta)] + \frac{1}{Fr}\left(1 - \frac{1}{\gamma}\right) + \text{M} \tag{14}$$
$$GG' = \in G'' - \beta[\,f + G\,] \tag{15}$$
$$\theta'' = \left(Pr(2f'\theta - f\theta') - \frac{2}{3}\frac{\beta}{1-\varphi}H[\theta_p - \theta] - \frac{1}{1-\varphi}PrE_c\beta H[F - f']^2 - PrE_cf''^2 - \frac{H(\eta)}{1-\varphi}MPrE_c\,F(\eta)\right) / (R_a + 1) \tag{16}$$
$$\theta_p''(\eta) = (2F\theta_p + G\theta_p' + \beta[\theta_p - \theta] + \beta EcPr[f' - F]^2 - \frac{3}{2}\epsilon EcPr[FF'' + (F')^2\,] - \frac{3}{2}MEcPrF(\eta)) / \left(\frac{\epsilon}{Pr} + \frac{3}{2}\frac{R_a}{\gamma}\right) \tag{17}$$

With boundary conditions

$$G'(\eta) = 0, f(\eta) = f_0, f'(\eta) = 1, F'(\eta) = 0, \theta(\eta) = 1\,, \theta_p' = 0\; as\; \eta \to 0$$
$$f'(\eta) = 0, F(\eta) = 0, G(\eta) = -f(\eta)\,, H(\eta) = \omega\,, \theta(\eta) \to 0\,, \theta_p(\eta) \to 0\; as\; \eta \to \infty \tag{18}$$

Where $f_0 = \frac{V_w}{(\vartheta C)^{1/2}}$ is the suction parameter and $Gr = \text{g}\frac{\beta^*(T_\omega - T_\infty)}{c^2x}$ is the local Grashof number.

Solution of the problem:

Here in this problem the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$ are not known but $f'(\infty) = 0, F(\infty) = 0, G(\infty) = -f(\infty), H(\infty) = \omega, \theta(\infty) = 0, \theta_p(\infty) = 0$ are given. We use Shooting method to determine the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$ .We have supplied $f''(0) = \propto_0$ and $f''(0) = \propto_1$. The improved value of $f''(0) = \propto_2$ is determined by utilizing linear interpolation formula. Then the value of $f'(\propto_2, \infty)$ is determined by using Runge-Kutta method. If $f'(\propto_2, \infty)$ is equal to $f'(\infty)$ up to a certain decimal accuracy, then $\propto_2$ i.e $f''(0)$ is determined, otherwise the above procedure is repeated with $\propto_0 = \propto_1$ *and* $\propto_1 = \propto_2$ until a correct $\propto_2$ is obtained. The same procedure described above is adopted to determine the correct values of $F(0), G(0), H(0), \theta'(0), \theta_p(0)$.

The essence of shooting technique to solve a boundary value problem is to convert the boundary value problem into initial value problem. In this problem the missing value of $\theta'(0)$ and $f''(0)$ for different set of values of parameter are chosen on hit and trial basis such that the boundary condition at other end i.e. the boundary condition at infinity (η_∞)are satisfied.A study was conducting to examine the effect of step size as the appropriate values of step size $\Delta\eta$ was not known to compare the initial values of $\theta'(0)$ and $f''(0)$ .If they agreed to about 6 significant digits, the last value of η_∞used was considered the appropriate value; otherwise the procedure was repeated until further change in η_∞did not lead to any more change in the value of $\theta'(0)$ and $f''(0)$ .The step size $\Delta\eta$ =0.1 has been found to ensure to be the satisfactory convergence criterion of 1×10^{-6}.The solution of the present problem is obtained by numerical computation after finding the infinite value for η. It has been observed from the numerical result that the approximation to $\theta'(0)$ and $f''(0)$ are improved by increasing the infinite value of η which is finally determined as η =5.0 with a step length of 0.1 beginning from η = 0.0 Depending upon the initial guess and number of steps N. the values of $f''(0)$ and $\theta'(0)$ are obtained from numerical computations which are given in table –2 for different parameters.

Table-1; Comparison results for the wall temperature gradient $-\theta'(0)$ in case of Ec=0,β=0,Ra=0 ,Gr=0, f_0=0

Pr	Chen	Grubka and Bobba	Able and Mahesha	G.K. Ramesh	Present study $-\theta'(0)$
0.72	1.0885	1.0885	1.0885	1.0886	1.0884
1.0	1.3333	1.3333	1.3333	1.3333	1.3332
10.0	4.7969	4.7969	4.7968	4.7968	4.7969

RESULTS AND DISCUSSION

The equations (12) to (17) with boundary conditions (18) were solved numerically, in double precision, by shooting method using the Runge-Kutta fourth order algorithm. The computations were done by the computer language FORTRAN-77.The results of heat transfer and skin friction coefficient characteristics are shown in Table-2, which shows that it is a close agreement with the existing literature. The effect of various parameters on the velocity profiles and temperature profiles also demonstrated graphically. In order to check the accuracy of our present numerical solution procedure used a comparison of wall temperature gradient -$\theta'(0)$ is made with those reported by with Chen[4],Grubka an Bobba[7], Able and Mahesha [13],G.K. Ramesh[6] for various values of Pradtl number Pr absence of other parameters which are given in table-1.Our present results are in a good agreement with the previous results.

Fig-2: Variation of up w.r.t Gr

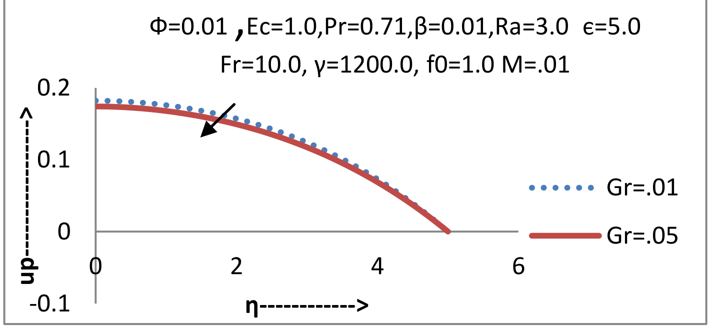


Fig-3: Variation of θp w.r.t Gr

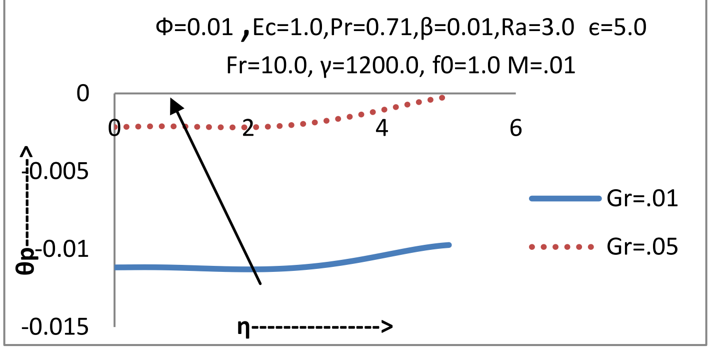


Fig-4: Variation of up w.r.t M

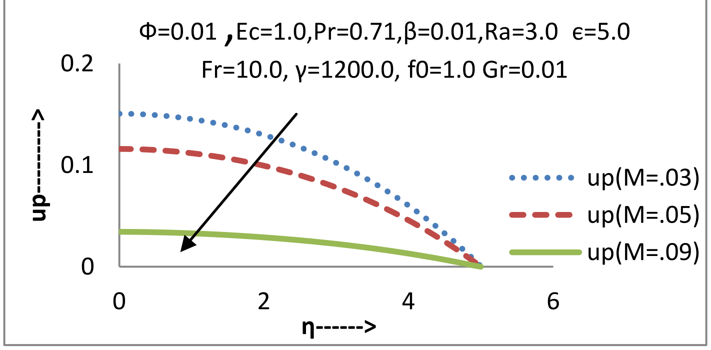
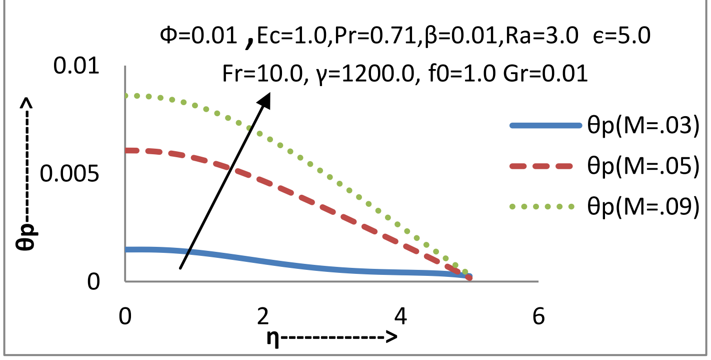


Fig-5: Variation of θp w.r.t M



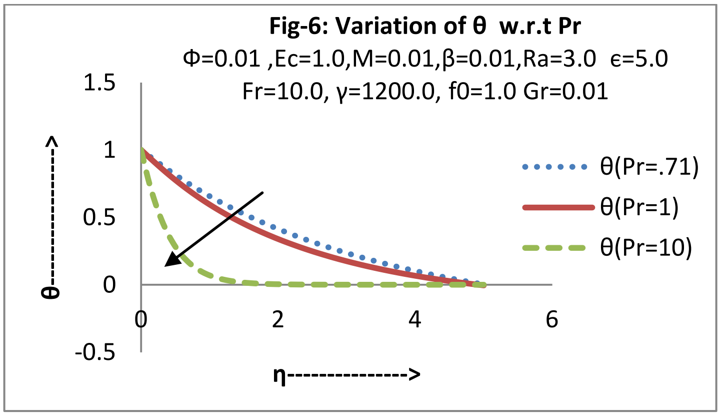


Fig-6: Variation of θ w.r.t Pr

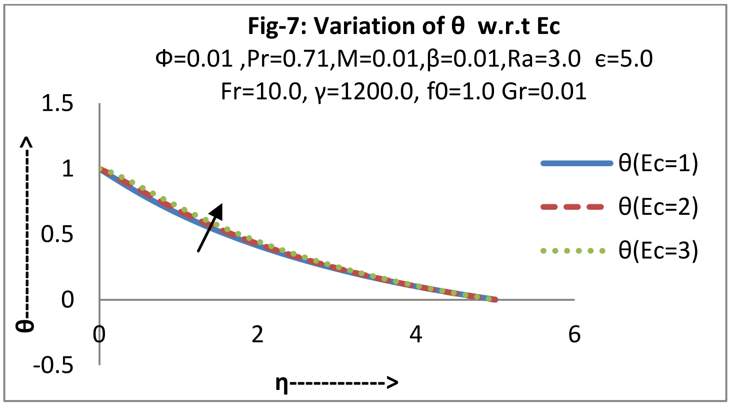


Fig-7: Variation of θ w.r.t Ec

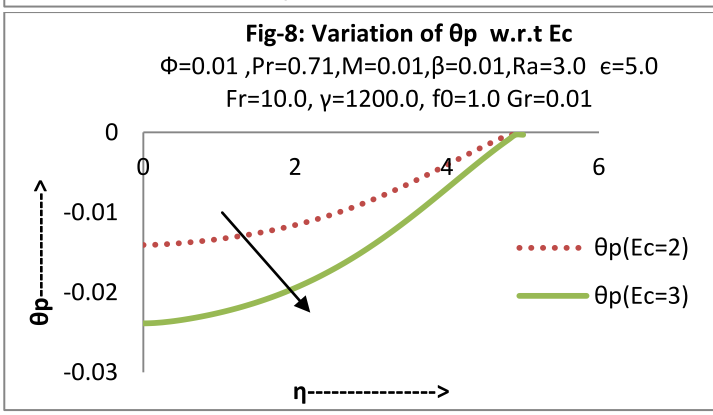


Fig-8: Variation of θp w.r.t Ec

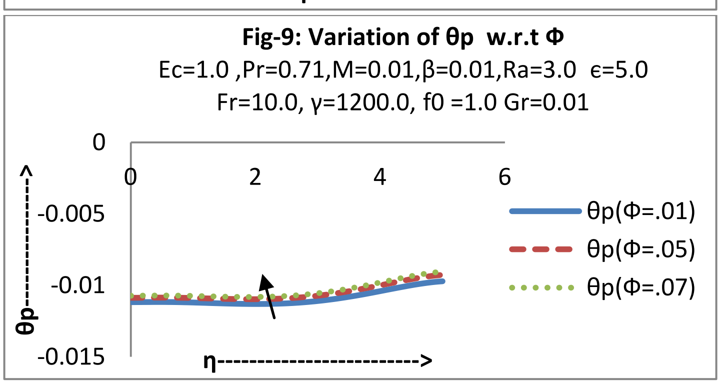


Fig-9: Variation of θp w.r.t Φ

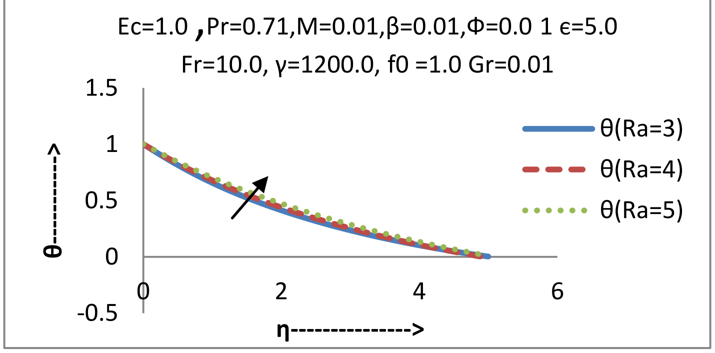


Fig-10: Variation of θ w.r.t Ra

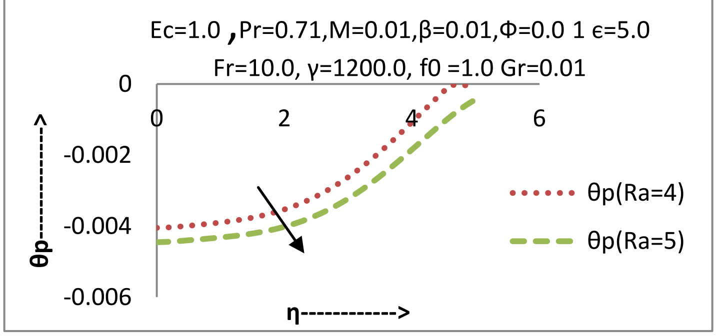


Fig-11: Variation of θp w.r.t Ra

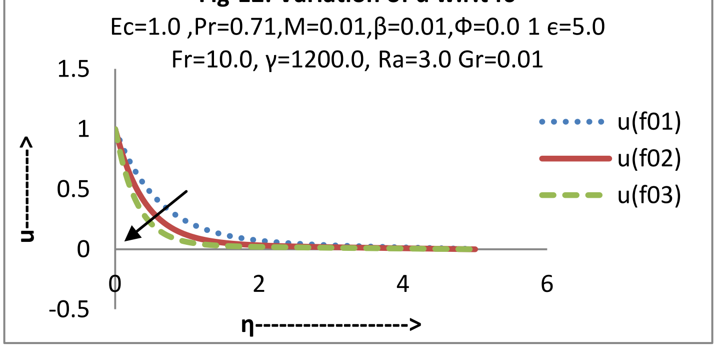


Fig-12: Variation of u w.r.t f0

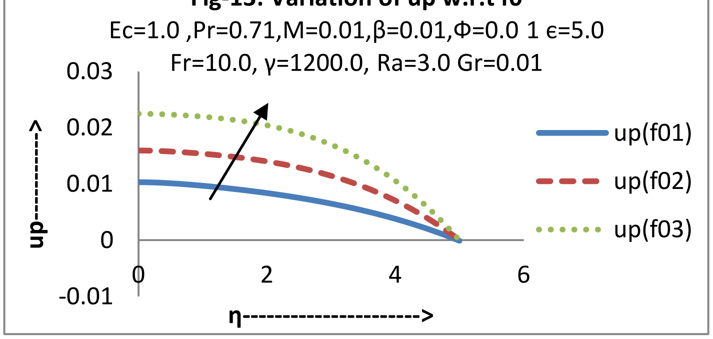
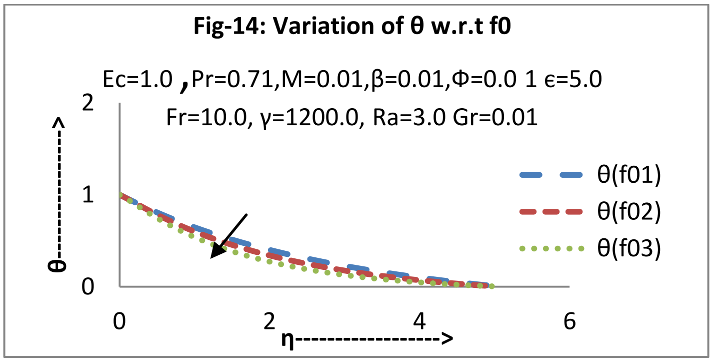
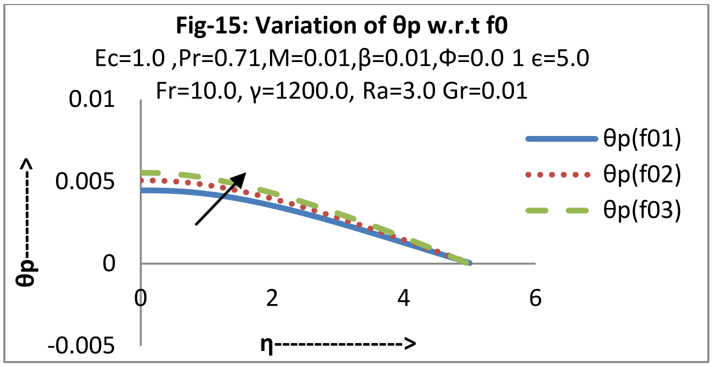
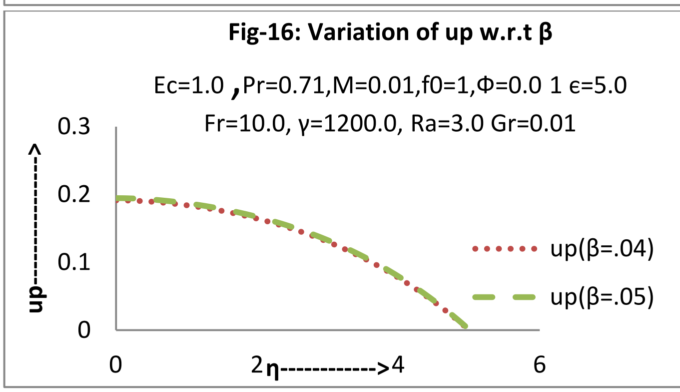


Fig-13: Variation of up w.r.t f0







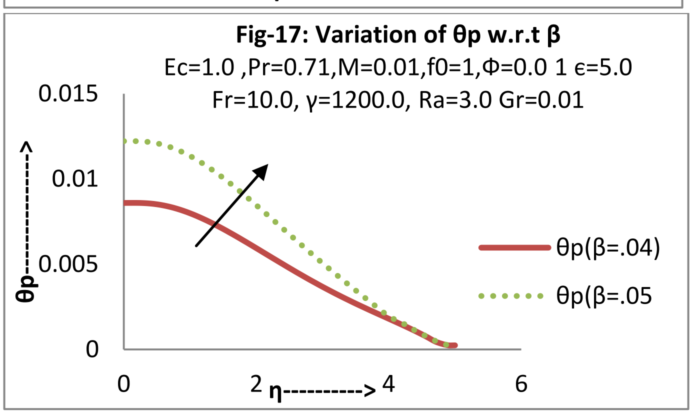


Figure-2&3 illustrate the velocity profiles u_p and temperature profiles θ_p versus η for various values of local Grashof number Gr .It is observed from the figures that the effect of increasing values of local Grashof number Gr is to decrease the velocity distribution u_p and increase the temperature distribution θ_p. Figures-4&5 illustrate velocity distribution u_p and temperature profiles θ_p with η for various values of electrical parameter M. It shows that velocity of dust phase decreases and temperature of fluid phase increases for increase value of electrical parameter M. Figure -6 depicts the variation in the temperature profiles θ for the selected values of Prandtl number Pr versus η. This figure indicates that the temperature θ decreases for increasing values of Pr.Figures.-7&8 depict the graph of fluid temperature θ and the dust temperature θ_p w.r.t. η for the selected values of Eckert number Ec. It is observed that the effect of increasing values of Ec, slightly increase in the temperature θ but the temperature θ_p decrease.Figure.-9 depicts the graph of temperature θ_p w.r.t η for various values of volume fraction φ . It is observed that the temperature θp slightly increases with increase of volume fraction φ.Figures-10&11, illustrate the variation of temperature profiles θ and θ_p of both phases versus η for the selected values of Ra It is observed that the fluid temperature θ slightly increases but dust temperature θ_p decreases for the of increasing values of Ra.Fig-12 & 14 depict the velocity profiles u & temperature profiles θ versus η for the effect of suction parameter f_0.Fluid phase velocity profiles u and temperature profiles θ decreases asymptotically for increasing value of suction parameter f_0.Fig-13 & 15 depict the velocity profiles u_p and temperature profiles θ_p versus η for the effect of suction parameter f_0 .Dust phase velocity profiles u_p and the temperature profiles θ_p increase with increase of suction parameter f_0. Fig.-16& 17 present the velocity distribution u_p and the temperature distribution θ_p with η for various values of fluid particle interaction parameter β. It is clearly observed from this figure -16 that no significant change in velocity u_p and from figure-17, the temperature θ_p increases for increasing value of β.

CONCLUSION

The free convective heat transport of a dusty fluid over a vertical permeable stretched sheet is examined in this work using numerical analysis. The energy equations now contain terms for thermal radiation, and all equations additionally include an electric component. Temperature and velocity profiles are visually shown and examined. The fluid particle interaction parameter, local Grashof number, suction parameter, radiation parameter, Prandtl number, Eckert number, electric parameter, and volume fraction are among the physical characteristics that have been discovered to have an impact on the issue under examination. On this basis of the above study we have the following observations:Velocity u_p of dust phase increases with increasing value of f_0 but decreases with increasing value of Gr and M.

1. There is no significant change in velocity u_p of dust phase as β increases.
2. Temperature of fluid decreases as Pr and f_0 increases but increases as Ra increases.
3. The temperature of fluid u slightly increases for effect of Ec.
4. For increasing value of f_0 the velocity of fluid phase decreases.
5. The temperature θ_p decreases for increasing value of Gr and φ but increases for increasing value of Ec, Ra. β, f_0 and M.
6. Both the skin friction and the rate of heat transfer increases for the increasing value ofβ, f_0 and M.
7. The rate of heat transfer decreases with increasing value of Ec & Ra but increases with increasing value of M, and Pr.
8. The skin friction decreases with increasing value of Pr but increases with increasing value of Ec, and Ra.
9. We have investigated the problem assuming the values $\varphi = 0.01, \epsilon = 5.0$,γ=1200.0 and $F_r = 10.0$.

In case of G.K. Ramesh the following results are obtained

1. The rate of heat transfer decreases with increasing value of β, f_0 Pr and Gr but increases with increasing value of Nr and Ec.
2. Radiation should be at its minimum in order to facilitate the cooling process.

TABLE-2; Values of wall velocity gradient $-f^{11}(0)$,temperature gradient $-\theta^{1}(0)$, F(0), -G(0), H(0) and $\theta_p(0)$ for different values of β, E_c , G_r , P_r ,M ,Ra , φ and f_0 where $\gamma = 1200, \epsilon$ =5

β	E_c	G_r	P_r	Ra	f_0	M	φ	$-f^{11}(0)$	F(0)	-G(0)	H(0)	$-\theta^{1}(0)$	θp(0)
0.03	1.0	0.01	0.71	3.0	1.0	0.01	.01	1.605152	0.188190	1.091958	0.132085	0.385504	0.007824
0.04								1.603132	0.191284	1.104388	0.131114	0.388778	0.008586
0.05								1.603280	0.194401	1.115960	0.129672	0.387538	0.012206
0.01	1.0	0.01	0.71	3.0	1.0	0.01	.01	1.608025	0.181953	1.06918	0.136667	0.383674	-0.011184
	2.0							1.602079	0182076	1.070127	0.131885	0.257831	-0.014057
	3.0							1.601691	0.182088	1.070294	0.131615	0.128644	-0.023888
0.01	1.0	0.01	0.71	3.0	1.0	0.01	.01	1.608025	0.181953	1.06918	0.136667	0.383674	-0.011184
		0.05						2.189575	.174077	1.053012	0.133249	0.220501	-0.002155
		0.07						1.535403	0.182504	1.071160	0.131646	0.409585	-0.004565
0.01	1.0	0.01	0.71	3.0	1.0	0.01	.01	1.608025	0.181953	1.06918	0.136667	0.383674	-0.011184
			1.0					1.608691	0.182132	1.069823	1.137367	0.471694	-0.016568
			10.0					1.611568	0.182009	1.069669	0.127543	2.310627	-0.280226
0.01	1.0	0.01	0.71	3.0	1.0	0.01	.01	1.608025	0.181953	1.06918	0.136667	0.383674	-0.0111

													84
				4.0				1.601506	0.181969	1.070321	0.131974	0.352027	-0.004047
				5.0				1.600657	0.182011	1.069978	0.130883	0.320693	-0.004449
0.01	1.0	0.01	0.71	3.0	1.0	0.01	.01	1.608025	0.181953	1.06918	0.136667	0.383674	-0.011184
					2.0			2.394478	0.185626	1.092548	0.131500	0.389793	-0.005415
					3.0			3.282218	0.189544	1.117087	0.131791	0.416650	-0.005415
0.01	1.0	0.01	0.71	3.0	1.0	0.03	.01	1.594789	0.150576	1.060289	0.141505	0.391182	0.001484
						0.05		1.585119	0.115945	1.049421	0.152503	0.393610	0.006072
						0.09		1.566027	0.0342118	1.023859	0.184870	0.404825	0.008615
0.01	1.0	0.01	0.71	3.0	1.0	0.1	.01	1.608025	0.181953	1.06918	0.136667	0.383674	-0.011184
							.05	1.607868	0.181952	1.069617	0.136704	0.383683	-0.010870
							.07	1.607770	0.181951	1.069617	0.136673	0.383687	-0.010729

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